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# C.U.SHAH UNIVERSITY Winter Examination-2018 

## Subject Name : Operations Research

Subject Code : 5SC01OPR1
Semester : 1 Date : 05/12/2018

Branch: M.Sc. (Mathematics)

Time : 02:30 To 05:30 Marks : 70

## Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

## SECTION - I

Q-1 a. Define term Operation Research.
b The pivotal element can be zero in simplex process. Determine whether statement is true or false.
c. What is the difference between slack variable and surplus variables?
d Define infeasible solution
e. Is Max $\mathrm{z}=80 \mathrm{x}+(100-0.01 \mathrm{y})$ y Linear Programming Problem?
f. Canonical and standard form of LPP is equal. Determine whether statement is true or false.
Q-2 a) A factory manufactures two articles A and B . To manufacture the article A , a certain machine has to be worked for 1.5 hours and in addition a craftsman has to work for 2 hours. To manufacture the article B, the machine has to be worked for 2.5 hours and in addition the craftsman has to work for 1.5 hours. In a week, the factory can avail of 80 hours of machine time and 70 hours of craftsman's time. The profit on each article A is Rs. 5 and that on each article B is Rs. 4. If all the articles produced can be solved away, find how many of each kind should be produced to earn the maximum profit per week. Formulate the problem as L.P. problem.
b) Use Penalty ( $\operatorname{Big} \mathrm{M}$ ) Method to solve following LP problem
$\operatorname{Max} z=6 x_{1}+4 x_{2}$
s.t. $2 \mathrm{x}_{1}+3 \mathrm{x}_{2} \leq 30$

$$
3 x_{1}+2 x_{2} \leq 24
$$

$$
x_{1}+x_{2} \geq 3
$$

$$
\text { and } x_{1}, x_{2} \geq 0
$$

## OR

Q-2 a) Vitamins $V$ and $W$ are found in two different foods $F_{1}$ and $F_{2}$. One unit of food $F_{1}$ contains 2 units of vitamin $V$ and 3 units of vitamin $W$. One unit of food $F_{2}$ contains 4 units of vitamin $V$ and 2 units of vitamin $W$. One unit of food $F_{1}$ and $\mathrm{F}_{2}$ cost Rs. 5 and 2.5 respectively. The minimum daily requirements (per a person) of vitamin V and W is 40 and 50 units respectively. Assuming that anything in excess of daily minimum requirement of vitamin V and W is not harmful, find out the optimal mixture of food $F_{1}$ and $F_{2}$ at the minimum cost which meets the daily minimum requirement of vitamins V and W . Formulate this as a linear programming problem.
b) Use graphical method to solve following LP problem
$\operatorname{Min} \mathrm{z}=3 \mathrm{x}_{1}+9 \mathrm{x}_{2}$
s.t. $\mathrm{x}_{1}+4 \mathrm{x}_{2} \leq 8$
$\mathrm{x}_{1}+2 \mathrm{x}_{2} \leq 4$
$x_{1}+4 x_{2}+6 x_{3} \leq 5$
and $\mathrm{x}_{1}, \mathrm{x}_{2} \geq 0$
Q-3 a) Write duel to following LP problem
i) $\operatorname{Max} z=-x_{1}+2 x_{2}$
ii) $\operatorname{Min} \mathrm{z}=7 \mathrm{x}_{1}+3 \mathrm{x}_{2}+8 \mathrm{x}_{3}$

$$
\begin{gathered}
\text { s.t. }-\mathrm{x}_{1}+3 \mathrm{x}_{2} \leq 10 \\
\mathrm{x}_{1}+3 \mathrm{x}_{2} \leq 6 \\
\mathrm{x}_{1}-\mathrm{x}_{2} \leq 2 \\
\text { and } \mathrm{x}_{1}, \mathrm{x}_{2} \geq 0
\end{gathered}
$$

$$
\text { s.t. } \quad 8 x_{1}+2 x_{2}+x_{3} \geq 3
$$

$$
3 x_{1}+6 x_{2}+4 x_{3} \geq 4
$$

$$
4 x_{1}+x_{2}+5 x_{3} \geq 1
$$

$$
x_{1}+5 x_{2}+2 x_{3} \geq 7 \text { and } x_{1}, x_{2}, x_{3} \geq 0
$$

b) Solve LPP by Simplex Method
$\operatorname{Max} \mathrm{z}=3 \mathrm{x}_{1}+2 \mathrm{x}_{2}$
s.t. $2 \mathrm{x}_{1}+\mathrm{x}_{2} \leq 5$

$$
\mathrm{x}_{1}+\mathrm{x}_{2} \leq 3
$$

and $\mathrm{x}_{1}, \mathrm{x}_{2} \geq 0$

Q-3 a) Use Duality Simplex Method, solve following problem
$\operatorname{Min} \mathrm{z}=2 \mathrm{x}_{1}+2 \mathrm{x}_{2}+4 \mathrm{x}_{3}$
s.t. $2 x_{1}+3 x_{2}+5 x_{3} \geq 2$
$3 \mathrm{x}_{1}+\mathrm{x}_{2}+7 \mathrm{x}_{3} \leq 3$
$\mathrm{x}_{1}+4 \mathrm{x}_{2}+6 \mathrm{x}_{3} \leq 5$
and $\mathrm{x}_{1}, \mathrm{x}_{2}, \mathrm{x}_{3} \geq 0$
b) Solve LPP by Simplex Method
$\operatorname{Max} z=70 x_{1}+150 x_{2}$
s.t. $4 x_{1}+6 x_{2} \leq 84$
$0.5 \mathrm{x}_{1}+\mathrm{x}_{2} \leq 60$
$\mathrm{x}_{1}, \mathrm{x}_{2} \geq 0$

## SECTION - II

Q-4 a. Mention difference between Transportation Model and assignment Model
b. The given assignment problem must be square. Determine whether statement is true or false.
c. Maximization problem can not be solved directly by transportation method. Determine whether statement is true or false.
d The solution to transportation problem with 3-rows and 3-columns is feasible if number of positive allocations are
e. Vogel's Approximation Method provides better optimum solution. Determine whether statement is true or false.
f. In an Assignment Programming Problem (APP), if number of rows and columns are equal then APP is said to be $\qquad$
Q-5 a) Solve minimal assignment problem whose effectiveness matrix is given below:

|  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | A | B | C | D |
| I | 45 | 40 | 51 | 67 |
| II | 55 | 40 | 61 | 53 |
| III | 49 | 52 | 48 | 64 |
| IV | 41 | 45 | 60 | 55 |
|  |  |  |  |  |

b) The paper manufacturing company has three warehouses located in 3 different areas, says A, B and C respectively. The company has to send from these warehouse to 3 destinations, says $\mathrm{D}, \mathrm{E}$ and F . The availability from warehouse A , B and C is 40,60 and 70 units. The demand at D, E and F is 70, 40 and 60 respectively. The transportation cost is shown in matrix (in Rs.).Use North-West Corner method to find out basic feasible solution.

|  |  | Markets (destinations) |  |  | $\begin{aligned} & \text { Supply } \\ & 40 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | E | F |  |
| Paper | A | 4 | 5 | 1 |  |


| units <br> (Sources) | B | 3 | 4 | 3 | 60 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | C | 6 | 2 | 8 | 70 |
|  | Demand | 70 | 40 | 60 | 170 |

Q-5 a) In the modification of a plant layout of a factory four new machines $\mathrm{M}_{1}, \mathrm{M}_{2}, \mathrm{M}_{3}$ and $\mathrm{M}_{4}$ are to be installed in a machine shop. There are five vacant places A, B, C, D and E available. Because of limited space, machine $\mathrm{M}_{2}$ can not be placed at $C$ and $M_{3}$ can not be placed at A. The cost of locating of machine $i$ to place $j$ in rupees is shown below. Find optimum assignment schedule.

| 倍 | A | B | C | D | E |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{M}_{1}$ | 9 | 11 | 15 | 10 | 11 |
| $\mathrm{M}_{2}$ | 12 | 9 | - | 10 | 9 |
| $\mathrm{M}_{3}$ | - | 11 | 14 | 11 | 7 |
| $\mathrm{M}_{4}$ | 14 | 8 | 12 | 7 | 8 |

b) Solve the following problem for basic feasible solution using Vogel's

Approximation Method:
Destinations

Sources

|  | A | B | C | D | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: |
| P | 1 | 2 | 1 | 4 | 20 |
| Q | 3 | 3 | 2 | 1 | 40 |
| R | 4 | 2 | 5 | 9 | 20 |
| S | 5 | 3 | 6 | 10 | 20 |
| Demand | 20 | 40 | 30 |  | 100 |

## Q-6 a) Use dual Simplex Method to solve LP problem

$\operatorname{Max} z=-3 x_{1}-2 x_{2}$
s.t. $x_{1}+x_{2} \geq 1$
$\mathrm{x}_{1}+\mathrm{x}_{2} \leq 7$
$\mathrm{x}_{1}+2 \mathrm{x}_{2} \geq 10 \quad \mathrm{x}_{1}, \mathrm{x}_{2} \geq 0$
b) Use UV (Modified Distribution Method) to following LP problem


Q-6 a) Use Wolf's method to solve following Quadratic Programming problem:
$\operatorname{Max} \mathrm{z}=4 \mathrm{x}_{1}+2 \mathrm{x}_{2}-\mathrm{x}_{1}{ }^{2}-\mathrm{x}_{2}{ }^{2}-5$
s.t. constraints $\mathrm{x}_{1}+\mathrm{x}_{2} \leq 4$,

$$
\mathrm{x}_{1}, \mathrm{x}_{2} \geq 0
$$

b) (i)The supply of each plant or factory, the demand of each warehouse and
transportation costs as per unit is sown in following table. Solve transportation problem by Least Cost Method.

| Plant/ factory | Warehouse |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{W}_{1}$ | $\mathrm{W}_{2}$ | ${ }_{3}$ | $\mathrm{W}_{4}$ | Supply |
| $\mathrm{F}_{1}$ | 21 | 16 | 25 | 13 | 11 |
| $\mathrm{F}_{2}$ | 17 | 18 | 14 | 23 | 13 |
| $\mathrm{F}_{3}$ | 32 | 27 | 18 | 41 | 19 |
| Demand | 6 | 10 | 12 | 15 | 43 |

(ii)Solve transportation problem by Column Minima Method

|  | Destinations |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Sources | 1 | 2 | 3 | Capacities |
|  | 2 | 2 | 3 | 10 |
|  | 1 | 2 | 15 |  |
| 1 | 3 | 5 | 40 |  |
| 20 | 15 | 30 |  |  |

